Appendix B: Vincenty's Inverse Solution

Notation:

a, b, major and minor semiaxes of the ellipsoid.

f, flattening = $(a - b) / a$.

ϕ, geodetic latitude, positive north of the equator.

L, difference in longitude, positive east.

s, length of the geodesic.

α_1, α_2, azimuths of the geodesic, clockwise from north; α_2 in the direction $P_1 P_2$ produced.

α, azimuth of the geodesic at the equator.

$u^2 = \cos^2\alpha \, (a^2 - b^2) / b^2$.

U, reduced latitude, defined by $\tan U = (1 - f) \tan \phi$.

λ, difference in longitude on an auxiliary sphere.

σ, angular distance $P_1 P_2$ on the sphere.

σ_1, angular distance on the sphere from the equator to P_1.

σ_m, angular distance on the sphere from the equator to the midpoint of the line.

Inverse Formula:

$$\lambda = L \textit{ (first approximation)} \tag{13}$$

$$\sin^2\sigma = (\cos U_2 \sin\lambda)^2 + (\cos U_1 \sin U_2 - \sin U_1 \cos U_2 \cos\lambda)^2 \tag{14}$$

$$\cos\sigma = \sin U_1 \sin U_2 + \cos U_1 \cos U_2 \cos\lambda \tag{15}$$

$$\tan\sigma = \sin\sigma / \cos\sigma \tag{16}$$

$$\sin\alpha = \cos U_1 \cos U_2 \sin\lambda / \sin\sigma \tag{17}$$

$$\cos 2\sigma_m = \cos\sigma - 2\sin U_1 \sin U_2 / \cos^2\alpha \tag{18}$$

λ is obtained by equations (10) and (11). This procedure is iterated starting with equation (14) until the change in λ is negligible.

$$s = bA(\sigma - \Delta\sigma), \tag{19}$$

where $\Delta\sigma$ comes from equations (3), (4), and (6):

$$A = 1 + \frac{u^2}{16384}\{4096 + u^2[-768 + u^2(320 - 175u^2)]\} \tag{3}$$

$$B = \frac{u^2}{1024}\{256 + u^2[-128 + u^2(74 - 47u^2)]\} \tag{4}$$

$$\begin{aligned}\Delta\sigma = B\sin\sigma\{&\cos 2\sigma_m + \frac{1}{4}B[\cos\sigma(-1 + 2\cos^2 2\sigma_m) \\ &- \frac{1}{6}B\cos 2\sigma_m(-3 + 4\sin^2\sigma)(-3 + 4\cos^2 2\sigma_m)]\}\end{aligned} \tag{6}$$

$$\tan\alpha_1 = \frac{\cos U_2 \sin\lambda}{\cos U_1 \sin U_2 - \sin U_1 \cos U_2 \cos\lambda} \tag{20}$$

$$\tan\alpha_2 = \frac{\cos U_1 \sin\lambda}{-\sin U_1 \cos U_2 + \cos U_1 \sin U_2 \cos\lambda} \tag{21}$$